The Schwarzschild metric is a tool that allows us to describe the relation between events in curvilinear space.

Why do we need a tool like this?

The motivation for this tool is that we want to be able to talk about the distance or time between events in a manner that is invariant for all observers. An invariant tool is important because this lets us handle events occurring at extremes greater than what we typically experience on Earth.

As we start talking about events that are occurring within lightseconds or lightminutes of each other, we need to use a framework that describes physics consistently at these extremes. This means Newtonian Mechanics with Galilean transforms are no longer sufficient. We can still use the basis of classical mechanics but if we use Lorentz transforms to transition between reference frames.

Essentially what this boils down to is: in everyday life, we can see events occurring and simply add velocities and when looking at the event from difference reference frames, we can agree that the physics is consistent. However, as these events occur at fractions the speed of light, velocities don’t simply add.

We can’t stand on a rocket moving half the speed of light and shine a flashlight and simply add these velocities. We’ve established that the speed of light is as fast as anything without mass can travel, and if we used what our everyday intuition tells us, we conclude that the light being emitted from the flashlight atop the rocket would be moving at 1.5 times the speed of light.

In these more extreme cases, which we observe when looking beyond Earth, we need to account for these types of things.

The main tool for this is in the framework of Special Relativity, called the spacetime interval. The spacetime interval gives us a formula for describing the distance between two events or the time separation between events in the same reference frame that all observers can agree on. The caveat here is that Special Relativity requires the reference frame to be in flat spacetime.

Why curved space?

The spacetime interval is invariant, which is what we want, but flat spacetime only exists locally in the reference frame. We use the General Relativity framework, which dictates that objects with mass distort the spacetime around them- curving it.

This part of the process isn’t as intense as fixing the approximations from Galilean to Lorentz transforms. Essentially, we just change coordinate systems and discuss the (reduced) radial distance from a massive object (that is curving spacetime) along with azimuthal and polar angles.

The product of this change, allows us to rewrite the simple spacetime interval from Special Relativity as the Schwarzschild Metric in General Relativity; which maintains the ability for us to describe the proper distance and time between events in a curved spacetime.

Things to be aware of:

The Schwarzschild metric reveals two singularities.

1. Coordinate Singularity

The coordinate singularity occurs when the proper distance away from a massive object is equal to its Schwarzschild radius:

This value causes division by zero:

This can be fixed by using other coordinate systems, in which, no longer becomes a problem. A list of alternative coordinate systems can be found at: <https://en.wikipedia.org/wiki/Schwarzschild_metric#Alternative_coordinates>

1. True Physical Singularity

The other singularity occurs at which . No coordinate system can fix this singularity. The gist is that, at , the curvature of spacetime becomes infinite and spacetime is no longer well defined. This was originally assumed to be a mathematical singularity, similar to the versions of Newton’s Law of Gravitation or Coulomb’s Law:

The difference being that, in the two above cases, can never truly equal . As objects with mass or charge do have some finite size to them. An example for both laws is consider the force of gravity and electrostatic force between two protons. A proton has a nonzero radius, meaning the maximum value for both forces is when:

The singularity inside the Schwarzschild surface, , does not have this restriction.

But what happens to the Schwarzschild Metric after you cross the discontinuity, when ?

Notably, outside the surface, when , we see the timelike component of the spacetime interval is satisfied by:

This is to say, that for any event to be the cause of another event (to be causally related), the timelike spacetime interval must be 0 or negative. However, when , we see that the sign on this component switches, as

Let such that:

We see that this changes the components of the spacetime interval such that is now the spacelike coordinate and

Becomes the timelike coordinate. But this means that the objects change in space () corresponds to movement in time. This is not true in the sense of time travel. This is true in the sense that radially inward corresponds to light that has fallen in before you and radially outward corresponds to light falling in after you crossed the Schwarzschild surface.

<https://youtu.be/KePNhUJ2reI>